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Compressibility Effects in Turbulent Shear Layers

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I. Introduction

FOR many reasons, it would be desirable to be able to make better predictions and to have a better understanding of the behavior of compressible turbulent shear layers. Such shear layers are important in the production of jet and rocket engine noise and are present in supersonic combustion jet engine designs. Compressible shear layers are also important in many high power laser systems.[1-3] The two-dimensional turbulent shear layer is well known to be strongly affected by compressibility.[4] In Sec. II of this Note, a Mach number M^+ is suggested which may be of value in correlating compressibility effects. In Sec. III, experimental results showing the decrease of shear layer width with increasing Mach number are compared with the corresponding variations of theoretical instability growth rates calculated by Blumen, et al.[5] In the final section, the significance of this comparison is discussed.

II. The Mach Number M^+

We denote velocity by u, density by ρ, specific heat ratio by γ, Mach number by M, and speed of sound by a. Subscripts *1* and *2* will refer to gases or conditions in the freestreams on the high- and low-speed sides of the shear layer, respectively. We define $\lambda_u = u_2/u_1$, $\lambda_\rho = \rho_2/\rho_1$, $\lambda_\gamma = \gamma_2/\gamma_1$, and $M_1 = u_1/a_1$. It seems reasonable to take the characteristic velocity to be $u_1 - u_2$. Since, in general, $a_1 \neq a_2$, there is a question as to the appropriate sound speed to be chosen. Two obvious candidates are the geometric and arithmetic average sound speeds. Instead of simply using one of these averages, we obtain M^+ from the following brief analysis.

It is assumed that there are certain large structures in the shear layer (this is known[6] to be the case for low-speed shear layers), and that these structures present similar profiles to the high- and low-speed streams and travel at a mean speed u_w. We estimate u_w by assuming that the total drag force on the large structures is zero, and therefore equate the dynamic pressure of the two freestreams with respect to the large structures, i.e.,

$$\left[1+\frac{\gamma_1-1}{2}\left(\frac{u_1-u_w}{a_1}\right)^2\right]^{\gamma_1/(\gamma_1-1)}-1$$
$$=\left[1+\frac{\gamma_2-1}{2}\left(\frac{u_w-u_2}{a_2}\right)^2\right]^{\gamma_2/(\gamma_2-1)}-1 \qquad (1)$$

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In support of Eq. (1), we note that in Brown and Roshko,[6] for $\lambda_u = 0.38$, $\lambda_\rho = 7$, and $M_1 \approx 0$, the mean observed speed of the large structures was $0.53u_1$. Using Eq. (1), $u_w = 0.55u_1$ is predicted. It is awkward, however, to obtain u_w from Eq. (1). Unless the Mach numbers are quite high and γ_1 and γ_2 are substantially different, Eq. (1) can be well approximated by

$$\rho_1(u_1-u_w)^2 = \rho_2(u_w-u_2)^2 \qquad (2)$$

Using Eq. (2) to obtain u_w, we can easily obtain the Mach numbers of the large structures with respect to the two freestreams. We find

$$(u_1-u_w)/a_1 = M_1(1-\lambda_u)/(1+\lambda_\rho^{-1/2})$$

and

$$(u_w-u_2)/a_2 = M_1(1-\lambda_u)/[(1+\lambda_\rho^{-1/2})\lambda_\gamma^{1/2}]$$

We take M^+ to be the geometric average of these two Mach numbers, i.e.,

$$M^+ = \frac{M_1(1-\lambda_u)}{(1+\lambda_\rho^{-1/2})\lambda_\gamma^{1/4}} \qquad (3)$$

Since λ_γ cannot normally be greater than 1.67 or less than 0.6, the difference between the geometric and arithmetic average values is very small.

III. Variation of Shear Layer Growth Rates with M^+, Theory and Experiment

Using M^+ as the correlation parameter, we compare the decrease in shear layer thickness with increasing Mach number observed experimentally with the corresponding decrease in the maximum linear instability growth rates from the theory of Blumen et al.[5] for a compressible, inviscid, constant density shear layer with a hyperbolic tangent velocity profile. The experimental data shown in Fig. 1 are taken from Brown and Roshko,[6] Maydew and Reed,[7] Ikawa and Kubota,[8] and Sirieix and Solignac[9] with the addition of the point at $M_1 = 5$ from Fig. 6 of Birch and Eggers.[4] The or-

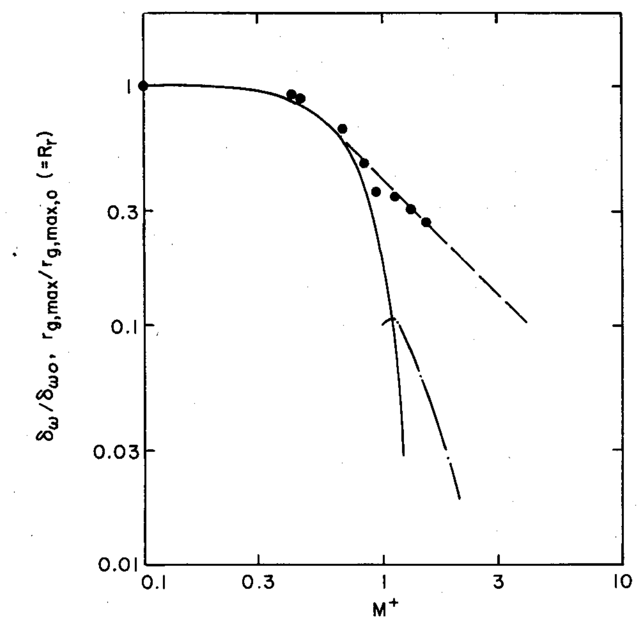

Fig. 1 Comparison of experimental and theoretical variations of shear layer growth rates with M^+. ●, experimental values of δ_ω, normalized to unity at $M^+ = 0$. Curves are maximum inviscid instability growth rates ($r_{g,max}$) calculated by Blumen et al.[5] for several types of instabilities, also normalized to unity at $M^+ = 0$.

dinate for the experimental data is $\delta_\omega/\delta_{\omega 0}$, where $\delta_\omega = \Delta u/[(\partial u/\partial y)_{\max}]$ is the vorticity thickness of the shear layer, Δu the velocity difference across the shear layer, $\partial u/\partial y$ the mean velocity gradient in the shear layer, and $\delta_{\omega 0}$ is δ_ω evaluated for $\lambda_\rho = 1$, $M_I \approx 0$. All of the experimental data are for $\lambda_u = 0$. The theoretical curves are the maximum instability growth rates from the theory of Blumen et al.[5] divided by the corresponding rates for $M^+ = 0$. Blumen et al. solve the temporal problem; their results have been transformed here to give spatial growth rates for $\lambda_u = 0$. $r_{g,\max} = [\alpha c_i/(1-c_r)]_{\max}$ in the notation of Blumen et al. The solid and dash-dot curves in Fig. 1 are for the modes of Blumen et al., with $c_r = 0$ and $c_r \neq 0$, respectively. [c_r is the propagation velocity of the instability with respect to a coordinate system moving at $(u_I + u_2)/2$.] Both these curves are for instabilities oriented normal to the freestream flow direction.

Because of the rapid decrease in the instability growth rates with increasing M^+, at higher values of M^+, modes oriented at an angle to the freestream flow direction may become important. This was pointed out earlier by Brown.[10] Since the equations of Blumen et al. are inviscid, the Squire transformation[5,11] can be used to obtain, from the basic results of Blumen et al., estimates for the effective growth rates for modes oriented at an angle to the freestream direction. When these latter modes are considered, it turns out that only modes derived from the results of Blumen et al. for $c_r = 0$ (solid curve, Fig. 1) are important, even for cases with $M^+ > 1$ Using this result, which we shall justify later, the maximum effective growth rate ratio ($R_r = r_{g,\max}/r_{g,\max,0}$) for cases with $M^+ > 0.8$, including modes oriented at an angle to the freestream direction, can be obtained from the basic results of Blumen et al. shown in Fig. 1 by a simple geometric construction. The effective M^+ for modes oriented at an angle θ to a flow with $M^+ = M_2^+$ is $M_I^+ = M_2^+ \cos\theta$. The effective growth rate ratio for these modes can be obtained by reading the value off the solid curve of Fig. 1 for $M_I^+ = M_2^+ \cos\theta$ and reducing this value by a factor $\cos\theta$ to account for the lower effective freestream velocity difference. This effective growth rate ratio can be obtained geometrically by drawing a straight line with slope -1 through the point on the solid curve of Fig. 1 at $M_I^+ = M_2^+ \cos\theta$ and reading off the growth rate ratio from this line at $M^+ = M_2^+$. For a fixed M_2^+, the largest effective growth rate ratios are obtained for modes oriented at an angle θ such that the line with slope -1 is tangent to the solid curve in Fig. 1. This line is shown dashed in Fig. 1. The maximum predicted effective growth rate ratios thus lie along this line for all M^+ greater than M^+ at the tangency point.

Although the transformation procedure for modes oriented at an angle to the freestream direction is slightly different for modes based on the results of Blumen et al. for $c_r \neq 0$, the growth rate ratios of these modes are sufficiently far below those for the modes with $c_r = 0$ that only the latter modes are important. With sufficient accuracy, the essential comparison is made by considering the relative position of lines with slope -1 tangent to the curves in Fig. 1 for $c_r = 0$ and $c_r \neq 0$.

IV. Discussion

The overall agreement between the theoretical and experimental results shown in Fig. 1 is quite good. This strongly suggests that an important factor contributing to the decrease in the shear layer growth rate with increasing M^+ is the decrease in the maximum growth rates of the large-scale Kelvin-Helmholtz instabilities. Thus it seems likely that linear instability theory can provide a powerful tool for the better understanding and prediction of compressible shear layer behavior.

It must, however, be pointed out that the theoretical results of Fig. 1 are for constant density shear layers, whereas the experimental data were taken with λ_ρ ranging from 1 (at $M^+ \approx 0$) up to about 5 (at the highest values of M^+). From Brown and Roshko's[6] Fig. 15, it can be estimated that for $\lambda_u = 0$, $M^+ \approx 0$, an increase of λ_ρ from 1 to 5 can, by itself, cause δ_ω to decrease by about 20%. In further studies of the relevance of the theoretical growth rates of large-scale instabilities to the measured growth rates of compressible shear layers, it would be desirable, in the theory, to consider cases with realistic variations of density across the shear layer.

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